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A TREATISE

ON

FRIENDLY SOCIETIES,

WITH

TABLES

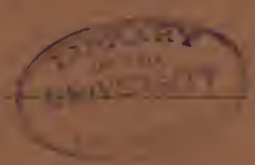
Constructed from the Sickness and Mortality experienced by Members of the Manchester Unity of Odd Fellows in passing through 1,821,418 years of life, being the largest experience hitherto collected of Friendly Societies in the United Kingdom.

BY

WILLIAM HATTON,

ACTUARY,

Brunswick Lodge, No. 118, M.U.O.F., and Court Little John, No. 2,782, A.O.F.



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PREFACE.

The following Report of the liability of each Lodge in the Brighton District of the Manchester Unity of Odd Fellows will explain the origin and the object of this Treatise, which is published in a separate form in the hope that it may be useful to and supply a want which has long been felt by Friendly Societies for their assistance and guidance:—

To the Valuation Committee of the Brighton District of the Manchester Unity of Odd Fellows.

Sirs and Brethren,

Agreeably to your instructions, I have made a valuation of the liabilities of each Lodge in the Brighton District, as on the 31st December, 1873, from a return from each Lodge in the District, showing the name of the Lodge; the number of members at each year of age; the contributions payable at each age to the sick and funeral fund; the amount of the sick and funeral fund; and the rate of interest realised on each investment.

The oldest Lodge in the District is the Brunswick Lodge, established on the 7th December, 1822. This Lodge, originally held at the Brunswick Arms, Brighton, for many years dispensed only sick *gifts* and funeral *donations*, each brother paying one penny per week to any sick brother, and one shilling towards any brother "that it may please God to call from our earthly Lodge." These collections proving financially inconvenient, a fixed equal, or rather unequal, contribution of four pence, and ultimately five pence per week per member, and fixed sick and funeral benefits were adopted. It was under this primitive plan of the mother Lodge of the same contribution for all ages that nineteen out of the twenty-seven Lodges in the District were afterwards established—viz., two in 1842, two in 1844, six in 1845, four in 1846, three in 1847, and two in 1853. When, however, the inadequacy and injustice of a low equal contribution for all ages were manifest, there was supplemented in 1853 a plan of additional annual contributions for *new* members exceeding twenty-two years of age. Lastly, chiefly in consequence of the publication in 1850 and 1862 of the Sickness and

Mortality Experience of the Order, contributions graduated according to age at entry became imperative for members admitted since the 1st August, 1866, by a General Rule, which also provided that Districts "shall have power to fix" the contributions of members admitted previous to that date. The Brighton District, and other Districts, having made this alteration in the contributions apply only to *new* members, the words *have power to* were struck out of the 38th General Rule at the Richmond A. M. C., 1874, thus making the alteration thereafter apply also to *all* members.

At the present time, therefore, there are, so far as contributions are concerned, three distinct classes of members in the District: (1) members who still pay, generally, five pence per week; (2) members who pay five pence per week and additional annual contributions; (3) members who pay contributions graduated according to age at entry.

The benefits assured to members in the District are generally 12s. 6d. per week in the first twelve months' sickness, 6s. 3d. per week in a continuance of the same sickness, £12 at the death of a member, and £8 at the death of a member's wife, being the same benefits as those assured when all members paid equally only five pence per week.

Before showing the net liability in expectation of each Lodge for sick and funeral benefits, it is necessary to say a few words with reference to the funeral liabilities.

The assurance of £12 payable at the death of a member, and the assurance of £8 payable at the death of a member's wife, called the funeral money, are liabilities not undertaken by the Lodge but by the District, in the following manner:—The twenty-seven Lodges comprising the Brighton District join together in paying the funeral claims of the District, thus spreading these claims over a larger number. The amount required is charged to each Lodge according to the number of its members, regardless of their ages at entry, by an unwise rule of the District. Thus, the youngest member admitted is made to contribute as much as the oldest member, which, of course, is not equitable, nor in accordance with the improved system of contributions graduated according to age at entry, as

a member admitted at age 18 should contribute only about one half of what should be contributed by a member admitted at age 45. But with the present practice of the District with regard to the funeral money, I have nothing more to do than to deal with the facts as I find them, and to value the liabilities of each Lodge accordingly.

The amount of the funeral claims of the District being, then, contributed by each Lodge in the District, according to the number of its members, regardless of their ages at entry, the value of £12 payable at the death of a member does not depend on the ages of the members of each Lodge, but on the average age of the members of the District. The age of the members of the District, one with another, being 34 years, the value of an assurance of £12, payable at the death of a person now aged 34, at 4 per cent., according to the Manchester Unity Table of Mortality for 1866-70, is £4·08552. Therefore, multiplying £4·08552 by 293, the number of members in the first Lodge—the Brunswick Lodge, in Class I., gives £1197·05736 = £1,197 1s. 2d., the value of all the members' funeral benefits in that class; and so on for the other classes and Lodges.

The value of all the wives' funeral benefits depends, in addition to the above, on (1) the number of wives and (2) the proportion of the age of the wife to that of the husband. With regard to the number of wives, the last reliable return to 31st December, 1871, shows that more than one-fourth of the members of the District were unmarried. Assuming, then, that three-fourths of the members are married, and that every member is of the same age with his wife, the value of an assurance of £8, payable at the death of a person now aged 34, at 4 per cent., according to the Manchester Unity Table of Mortality for 1866-70, is £2·72368. Therefore, multiplying £2·72368 by 220, the number of wives in the Brunswick Lodge, in Class I., gives £599·2096 = £599 4s. 2d., the value of all the wives' funeral benefits in that class; and so on for the other classes and Lodges.

These values of the average funeral benefits being liabilities undertaken by the District for corresponding amounts contributed by the Lodge out of the members' contributions, the

actual funeral benefits of the Lodge may be greater or less, but these are the amounts expected to be charged to the Lodge by the District for paying all the funeral claims of the District.

With these explanations, I will now proceed to show the net liability and amount of the sick and funeral fund of each Lodge in the District as on the 31st December, 1873:—

Name of Lodge.	Established.	Net Liability.			Sick & Funeral Fund.			Surplus.			Deficiency.		
		£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.
Brunswick	1822	7,375	14	0	3,451	1	7				3,924	12	5
Western Star	1842	5,534	16	1	3,858	7	4				1,676	8	9
Duke of Norfolk	,,	3,735	16	7	1,616	0	0				2,119	16	7
Weald of Sussex	1844	1,982	16	1	1,589	16	5				392	19	8
Prince of West Sussex	,,	2,068	0	10	1,904	12	9				163	8	1
Tilgate Forest	1845	1,693	15	4	1,960	11	8	266	16	4			
City	,,	5,049	3	5	3,930	4	0				1,118	19	5
Victoria	,,	2,920	18	1	3,794	14	10	873	16	9			
Mountain	,,	727	8	7	359	8	3				368	0	4
Saint Peter's	,,	1,716	10	4	938	5	5				778	4	11
Beulah	,,	4,017	13	6	2,923	9	3				1 094	4	3
Waterloo	1846	1,865	10	4	531	4	7				1 334	5	9
Oak and Ivy	,,	1,181	17	5	1,120	18	3				60	19	2
Bee Hive	,,	2,151	19	2	2.303	1	0	151	1	10			
Myrtle	,,	1,761	17	9	796	5	3				965	12	6
Hall of Justice	1847	835	5	2	639	6	5				195	18	9
Olive Branch	,,	2,382	18	2	1,323	2	11				1 059	15	3
Castle	,,	2,832	2	10	1.763	3	0				1,068	19	10
Hayward's Heath	1853	1,688	15	7	1,210	14	1				478	1	6
Cliftonville	,,	758	9	5	770	3	8	11	14	3			
York	1859	620	5	8	639	10	6	19	4	10			
Hanover	1861	411	10	7	275	3	9				136	6	10
Star of Hope	,,	1,031	0	7	382	15	9				648	4	10
Deerswood	1864	837	4	7	337	0	1				500	4	6
Rose	1865	724	18	1	517	0	2				207	17	11
Albert Victor	,,	173	10	10	133	3	2				40	7	8
Prince Alfred	1866	300	1	2	251	5	7				48	15	7
		56,380	0	2	39,320	9	8	1,322	14	0	18,382	4	6

This valuation, in order to determine as to the solvency of each Lodge in the District, has been made on the basis of the Sickness and Mortality Experience of the Manchester Unity for 1866-70, at the average rate of interest realised by each Lodge, up to 4½ per cent. No less than twenty-one out of the twenty-seven Lodges in the District—in fact all the older Lodges, with but one exception, a country Lodge—realise, on an average, a higher rate of interest than 3 per cent., and the investments on Mortgages, Town Debentures, &c., are such as are most likely to realise the same rates of interest in the future.

The above total deficiency of £18,382 4s. 6d. in some of the Lodges in the District has arisen from the inadequacy of the weekly contributions of the members admitted previous to the 1st August, 1866, as will be seen by comparing, in the valuation, the value of each class of members' contributions to each Lodge in the District with the value of each class of members' sick and funeral benefits.

It being now compulsory, by the 38th General Rule, for the District to fix the amount of contributions to be paid by the members of each Lodge in the District, admitted previous to the 1st August, 1866, the good sense and right feelings of the District Officers and Deputies will doubtless guide this difficult affair to a satisfactory solution.

As there is no good text-book to which I could refer the District Officers and Deputies for assistance in the matter, I have drawn up a brief Treatise on Friendly Societies, explaining the modern methods of constructing tables of life and health contingencies, which I have added by way of supplement to this Report.

The Preparatory Tables, involving the rate of interest as well as the rates of sickness and mortality, were constructed by the aid of logarithms. They could have been constructed by actual multiplication, but the process would have been more laborious and the risk of error greater. For the purposes of Friendly Societies five figures of the tabular values being sufficient, I have used Bremiker's table of five figures of numbers to six of logarithms. This work, uniform with this Treatise, may be procured through Messrs. C. and E. Layton, 150 Fleet Street, at the low price of two shillings per copy in paper cover.

In the following Treatise, which is complete as far as it goes, I have endeavoured to make the language accurate and the computations free from errors, typographical or otherwise.

By means of this Treatise may be constructed tables of weekly contributions by simple multiplication and division.

Example 1. Weekly contribution, at age 18, for an assurance of 12s. 6d. per week in the first twelve months' sickness during life, 6s. 3d. per week in a continuance of th

same sickness, £12 at the death of a member, and £8 at the death of a member's wife.

Table VIII.	1·2724	×	·625	=	·79525	
,, X.	·5474	×	·3125	=	·17106	
,, V.	·01454	×	12	=	·17448	
,, V.	·01454	×	8	=	·11632	

52) 1·25711 (·0241 = 6d.
1 04
217
208
91

This example shows that the early contribution of 5d. per week for an assurance of these benefits was an inadequate one, even at the youngest age at entry, under the plan of the same contribution for all ages, from age 18 to 36.

Example 2. Weekly contribution, at age 36, for an assurance of 12s. 6d. per week in the first twelve months' sickness during life, 6s. 3d. per week in a continuance of the same sickness, £12 at the death of a member, and £8 at the death of a member's wife.

Table VIII.	1·9730	×	·625	=	1·23313
,, X.	1·2538	×	·3125	=	·39181
,, V.	·02484	×	12	=	·29808
,, V.	·02484	×	8	=	·19872

52) 2·12174 (·0408 = 10d.
2 08
417

This example shows that the early contribution of 5d. per week for an assurance of these benefits was exactly only half the sum which should have been paid at the oldest age, under the plan of the same contribution for all ages, from age 18 to 36.

It being then clear, both from the valuation and the supplementary Treatise, that the contributions of the old members have been long inadequate, the old members cannot object to escaping, should the District Officers and Deputies decide on their paying only for the future, what they should have paid during the whole time of their membership.

Yours truly and fraternally,

WILLIAM HATTON, Actuary,
Brunswick Lodge, No. 118.

30 New Road, Brighton,
August 20th, 1874.

A TREATISE ON FRIENDLY SOCIETIES.

I. PROBABILITY AND THE AVERAGE DURATION OF LIFE.

1. If a be the number of ways in which an event can happen, and b the number of ways in which it can fail, and all these ways are equally possible, the probability of the event happening is $\frac{a}{a+b}$, and the probability of its failing is $\frac{b}{a+b}$.

2. The probability of the continuance of life of one person is very uncertain, and differs under different circumstances in persons of the same age. It is greater in healthy than in unhealthy persons; in persons living in a healthy climate or locality than in persons living in an unhealthy one; in persons living in the country than in persons living in a large town or city; in persons following healthy or safe occupations than in persons following unhealthy or dangerous ones; in females than in males; and in married persons than in single persons. For general purposes it is sufficient to find the average probability out of a large number of unselected persons of the same age. This average probability is found by a Table of Mortality showing, from experience, out of a large number of unselected persons of the same age, the number dying in each succeeding year of age, and, consequently, the number living at each successive age. It is necessary to assume, that out of any other equal number of persons of the same age and class the number living at each age would be nearly the same, if they live under similar circumstances.

3. The Tables of Mortality of the members of Friendly Societies are not very numerous. They comprise:

(a) The Friendly Societies' Table, constructed by Mr. Ansell, from returns obtained by the Society for the Diffusion of Useful Knowledge from Friendly Societies in England for 1823-27, embracing the experience of 24,323 years of life.*

(b) The Friendly Societies' Table, constructed by Mr. Neison, from the returns of Friendly Societies in England and Wales for 1836-40, sent to the Registrar, embracing the experience of 1,147,143 years of life.†

(c) The Odd Fellows', Manchester Unity Table, constructed by Mr. Ratcliffe, from returns of the Lodges composing the Unity for 1846-48, embracing the experience of 621,561 years of life.‡

(d) The Friendly Societies' Table, constructed by Mr. Finlaison, from the returns of Friendly Societies in England and Wales for 1846-50, sent to the Registrar, embracing the experience of 793,759 years of life.§

(e) The Odd Fellows', Manchester Unity Table, constructed by Mr. Ratcliffe, from returns of the Lodges composing the Unity for 1856-60, embracing the experience of 1,006,272 years of life.¶

(f) The Odd Fellows', Manchester Unity Table, constructed by Mr. Ratcliffe, from returns of the Lodges composing the Unity for 1866-70 (see Table I., page 5), embracing the experience of 1,321,048 years of life, being the largest experience hitherto collected of Friendly Societies in the United Kingdom.‖

4. A comparative view of these tables would be useful, but would be foreign to the object of this Treatise.

* *A Treatise on Friendly Societies.* By Charles Ansell, Esq., F.R.S., 1835, p. 119.

† *Contributions to Vital Statistics.* By F. G. P. Neison, F.L.S., 1845, p. 28.

‡ *Observations on the Rate of Mortality and Sickness existing amongst Friendly Societies.* By Henry Ratcliffe, 1850, p. 24.

§ *Return of the Friendly Societies' Sickness and Mortality.* Mr. Alexander Glen Finlaison's Report. Ordered by the House of Commons to be printed, 16th August, 1853, p. 3.

¶ *Observations on the Rate of Mortality and Sickness existing amongst Friendly Societies.* By Henry Ratcliffe, 1862, p. 25.

‖ *Supplementary Report, July 1st*, 1872, of the Independent Order of Odd Fellows, Manchester Unity Friendly Society, p. 20.

5. Let l_x denote the number of persons living at the age x according to any mortality table, ${}^{n}l_x$ the number living at the age n years older than x, and ${}^{-n}l_x$ the number living at the age n years younger than x; then, by Art. 1, the probability of a person whose age is x living n years is

$$\frac{{}^{n}l_x}{l_x}.$$

Example 1. Probability of a person aged 18 living 7 years, or attaining the age of 25, according to the Manchester Unity Table of Mortality for 1866-70.

$$\frac{{}^{7}l_{18}}{l_{18}}=\frac{l_{25}}{l_{18}}=\frac{95711}{100000}=\cdot95711$$

6. Of l_x persons living at the age x, $l_x-{}^{n}l_x$ is the number dying between the ages x and $x+n$, or within n years; then, by Art. 1, the probability of a person whose age is x dying within n years is

$$\frac{l_x-{}^{n}l_x}{l_x}.$$

Example 2. Probability of a person aged 18 dying within 7 years, or before attaining the age of 25, according to the Manchester Unity Table of Mortality for 1866-70.

$$\frac{l_{18}-l_{25}}{l_{18}}=\frac{100000-95711}{100000}=\frac{4289}{100000}=\cdot04289$$

7. The ratio of the number of persons dying within a year to the number living at the beginning of the year is called the *yearly rate of mortality*. Thus, of 100,000 persons living at the age 18, 560 die within a year. Therefore $560 \div 100000 \times 100 = \cdot5600$, is the yearly mortality per cent., according to the Manchester Unity Table of Mortality for 1866-70.*

8. The average number of years which persons of the same age, one with another, live according to the given mortality table, is called the *expectation, or average duration, of life.*

* The number dying in each year of age being expressed in the Manchester Unity Table of Mortality by a whole number, decimal omitted, the correct number, including decimal, may be found by multiplying the mortality per cent. for each age by the number living at each age, and then dividing by 100. Thus the number dying in the 20th year of age is $5817 \times 99440 \div 100 = 578{\cdot}442480$, and so on. The decimal must be supplied as in column *d* of Table I. in calculating the rate of mortality per cent. for each age.

Let $\mathring{e}_x$ denote the expectation, or average duration, of life of a person of the age x, z the number of years from age x to the oldest age attained by any life according to the table, and let us suppose that the persons dying in each year die at equal intervals therein. Then of l_x persons living at the age x, $l_x - {}^1l_x$ die in the first year, living among them $\frac{1}{2}(l_x - {}^1l_x)$ years, while those who attain the age $x+1$ live ${}^1l_x$ whole years; therefore the l_x persons live in the first year, $\frac{1}{2}(l_x - {}^1l_x) + {}^1l_x = \frac{1}{2}(l_x + {}^1l_x)$ years. Similarly l_x persons live through life $\frac{1}{2}(l_x + {}^1l_x) + \frac{1}{2}({}^1l_x + {}^2l_x) + \ldots + \frac{1}{2}({}^{z-1}l_x + {}^zl_x) + \frac{1}{2}{}^zl_x = \frac{1}{2}l_x + {}^1l_x + {}^2l_x + \ldots + {}^zl_x$ years, which, divided by l_x, gives

$$\mathring{e}_x = \tfrac{1}{2} + \frac{{}^1l_x + {}^2l_x + \ldots + {}^zl_x}{l_x} \quad \ldots\ldots (1).$$

This demonstrates the form of the rule.

The rule is, *divide the sum of the numbers living at all the older ages by the number living at the given age, and then add half a year to the quotient.*

The number in column L of Table I., opposite each age, is the sum of the numbers living at all the older ages; thus

$$\mathrm{L}_x = {}^1l_x + {}^2l_x + {}^3l_x + \ldots \text{ to the last tabular age;}$$
$$\mathrm{L}_x = {}^1l_x + {}^1\mathrm{L}_x;$$

therefore

$$\mathring{e}_x = \frac{\mathrm{L}_x}{l_x} + \tfrac{1}{2} \quad \ldots\ldots (2).$$

By means of column L we are now enabled to compute the average duration of life by simple division.

Example 3. Average duration of life of a person aged 18, according to the Manchester Unity Table of Mortality for 1866-70.

$$\frac{\mathrm{L}_{18}}{l_{18}} + \tfrac{1}{2} = \frac{4237040}{100000} + \tfrac{1}{2} = 42{\cdot}37 + {\cdot}5 = 42{\cdot}87 \text{ years}$$

TABLE I. *Rate of Mortality and the Average Duration of Life among the Members of the Independent Order of Odd Fellows, Manchester Unity Friendly Society for* 1866-70, *in the Rural, Town, and City Districts combined.*

Age. x	Living. l_x	Dying. d_x	Mortality per Cent.	Sum of the Numbers Living at all the older Ages. L_x	Average Duration of Life. $\mathring{e}_x$
18	100000	560·000000	·5600	4237040	42·87
19	99440	578 442480	·5817	4137600	42·11
20	98862	596·533308	·6034	4038738	41·35
21	98266	614·260766	·6251	3940472	40·60
22	97652	631·222528	·6464	3842820	39·85
23	97021	647·518154	·6674	3745799	39·11
24	96374	663·053120	·6880	3649425	38·37
25	95711	678·016724	·7084	3553714	37·63
26	95033	692·125339	·7283	3458681	36·89
27	94341	705·010293	·7473	3364340	36·16
28	93636	716·596308	·7653	3270704	35·43
29	92920	727·099000	·7825	3177784	34·70
30	92193	736·345491	·7987	3085591	33·97
31	91457	744·368523	·8139	2994134	33·24
32	90713	756 546120	·8340	2903421	32·51
33	89957	772·910544	·8592	2813464	31·78
34	89184	793·113312	·8893	2724280	31·05
35	88391	817·174795	·9245	2635889	30·32
36	87574	844·738804	·9646	2548315	29·60
37	86729	871·799908	1·0052	2461586	28·88
38	85858	897·387816	1·0452	2375728	28·17
39	84960	922·580640	1·0859	2290768	27·46
40	84037	946·760842	1·1266	2206731	26·76
41	83090	970·075750	1·1675	2123641	26·06
42	82119	993·968376	1·2104	2041522	25·36
43	81125	1018·443250	1·2554	1960397	24·67
44	80107	1043·313568	1·3024	1880290	23·97
45	79064	1068·549960	1·3515	1801226	23·28
46	77996	1093·971896	1·4026	1723230	22·59
47	76902	1125·691476	1·4638	1646328	21·91
48	75777	1163·252727	1·5351	1570551	21·23
49	74614	1206·135310	1·6165	1495937	20·55
50	73408	1253·735232	1·7079	1422529	19·88
51	72154	1305·482322	1·8093	1350375	19·22
52	70849	1357·821085	1·9165	1279526	18·56
53	69491	1410·111372	2·0292	1210035	17·91
54	68081	1462·243718	2·1478	1141954	17·27
55	66619	1513·583680	2·2720	1075335	16·64
56	65106	1563·715908	2·4018	1010229	16·02
57	63542	1625·785612	2·5586	946687	15·40
58	61916	1697·860552	2·7422	884771	14·79
59	60218	1778·117104	2·9528	824553	14·19

Rate of Mortality and Average Duration of Life Tables—(continued).

Age. x	Living. l_x	Dying. d_x	Mortality per Cent.	Sum of the Numbers Living at all the older Ages. L_x	Average Duration of Life. $\mathring{e}_x$
60	58440	1864·352880	3·1902	766113	13 61
61	56576	1954·361344	3·4544	709537	13·04
62	54622	2032·102266	3·7203	654915	12·49
63	52589	2097·196731	3·9879	602326	11·95
64	50492	2149·545424	4 2572	551834	11·43
65	48343	2189·116069	4·5283	503491	10·91
66	46154	2215·853540	4·8010	457337	10·41
67	43938	2261·400984	5·1468	413399	9.91
68	41677	2319 533435	5·5655	371722	9·42
69	39358	2383·992776	6·0572	332364	8·94
70	36974	2448·011566	6·6209	295390	8·49
71	34526	2506·414970	7·2595	260864	8·06
72	32020	2510 656180	7·8409	228844	7·65
73	29510	2527 029830	8·5633	199334	7·25
74	26983	2490·503917	9·2299	172351	6·89
75	24493	2426·227594	9·9058	147858	6·54
76	22067	2337·160104	10·5912	125791	6 20
77	19730	2236 790100	11·3370	106061	5·88
78	17493	2124·209976	12 1432	88568	5·56
79	15369	1999·476162	13·0098	73199	5 26
80	13374	1863 921006	13·9369	59825	4 97
81	11510	1717 798440	14·9244	48315	4·70
82	9792	1565·799552	15·9906	38523	4·43
83	8226	1409·574456	17·1356	30297	4·18
84	6817	1257·013898	18·4394	23480	3·94
85	5560	1093·207200	19·6620	17920	3·72
86	4467	940·004211	21·0433	13453	3·51
87	3527	792·135984	22 4592	9926	3·31
88	2735	653·943970	23·9102	7191	3·13
89	2081	528·486598	25·3958	5110	2·96
90	1553	418·007033	26·9161	3557	2·79
91	1135	323·146985	28·4711	2422	2·63
92	812	244·233360	30·0780	1610	2·48
93	568	180·265024	31·7368	1042	2 33
94	388	129·776683	33·4476	654	2·19
95	258	90·842574	35·2103	396	2·03
96	167	61·831583	37 0249	229	1·87
97	105	40·833765	38 8893	124	1·68
98	64	26·113216	40·8019	60	1·44
99	38	16·163794	42·5363	22	1·08
100	22	9·850032	44·7731		·50

As the returns for 1866-70 include few members after age 83, from that age the rate of mortality for England and Wales, according to the 12th Report of the Registrar General, pages 75 and 79, with slight modifications for locality, has been adopted in the *Supplementary Report*.

II. LIFE ANNUITIES.

9. When an annuity (or annual payment) is payable during life, it is called a *life annuity*.

10. *To find the value of an endowment of* £1 *payable in* n *years if* x *be then alive.*

By Art. 5, the probability of a person whose age is x living n years is $\frac{{}^{n}l_x}{l_x}$.

Let $v^n = \frac{1}{(1+i)^n}$ denote the present value of £1 due n years hence; then the present value of £1 payable in n years if x be then alive is

$$\frac{{}^{n}l_x v^n}{l_x} \quad \ldots\ldots (1).$$

Example 1. Value of an endowment of £10 payable if a person aged 18 attain the age of 25, at 3 per cent., according to the Manchester Unity Table of Mortality for 1866-70.

$$\frac{l_{25}v^7}{l_{18}} \times 10 = \frac{95711 \times \cdot 81309}{100000} \times 10 = 7\cdot 7822$$

By logarithms,

$$\begin{aligned} \log l_{25} &= 4\cdot 980962 \\ \log v^7 &= \bar{1}\cdot 910139 \\ \text{co log } l_{18} &= \bar{5}\cdot 000000 \\ \log 10 &= 1\cdot 000000 \\ \log \quad &= 0\cdot 891101 \\ &= 7\cdot 7822 \end{aligned}$$

In formula (1) the numerator of the fraction is the present value of the number of pounds payable in n years, and the denominator the number now living to contribute equally the present fund.

If (1) be transformed by multiplying both numerator and denominator by v^x, by which the value of the fraction is not altered, it becomes

$$\frac{{}^{n}l_x v^{x+n}}{l_x v^x} \quad \ldots\ldots (2),$$

which is a more convenient formula for constructing tables of the values of annuities (presently to be explained) than formula (1).

The number in column D of Table II., opposite each age, is the product of the number living at that age into the present value of £1 due as many years hence as are equal to that age, so that in (2) the numerator of the fraction is equal to D_{x+n}, and the denominator to D_x. Hence the formula becomes finally

$$\frac{D_{x+n}}{D_x} \quad \ldots\ldots (3).$$

By means of colum D we are now enabled to compute the values of endowments by simple division.

For example, as above: Value of an endowment of £10 payable if a person aged 18 attain the age of 25, at 3 per cent.

$$\frac{D_{25}}{D_{18}} \times 10 = \frac{45712}{58739} \times 10 = 7{\cdot}7822$$

11. *To find the value of an annuity of* £1 *due at the end of every year through which* x *shall live.*

The value of the annuity is the sum of the present values of £1 due at the end of 1, 2, 3, . . . years, to the last tabular age, if x shall live so long.

Let a_x denote the present value of an annuity of £1 due at the end of every year through which x shall live; then, by Art. 10,

$$a_x = \frac{{}^1l_x v}{l_x} + \frac{{}^2l_x v^2}{l_x} + \ldots + \frac{{}^z l_x v^z}{l_x}$$

$$= \frac{{}^1l_x v + {}^2l_x v^2 + \ldots + {}^z l_x v^z}{l_x} \quad \ldots\ldots (4).$$

In formula 4 the numerator of the fraction is the sum of the present values of the number of pounds due at the end of 1, 2, 3, . . . years, to the last tabular age, and the denominator the number now living to contribute equally the present fund.

If (4) be transformed by multiplying both numerator and denominator by v^x, by which the value of the fraction is not altered, the equation becomes

$$a_x = \frac{{}^1l_x v^{x+1} + {}^2l_x v^{x+2} + \ldots + {}^z l_x v^{x+z}}{l_x v^x} \quad \ldots\ldots (5).$$

The number in column N of Table II., opposite each age, is the sum of the numbers in column D opposite all the older ages;* thus

$$N_x = D_{x+1} + D_{x+2} + D_{x+3} + \ldots \text{ to the last tabular age;}$$
$$N_x = D_{x+1} + N_{x+1};$$

therefore
$$a_x = \frac{N_x}{D_x} \ldots\ldots (6).$$

Example 2.—Value of an annuity of £1 on the life aged 18.

$$\frac{N_{18}}{D_{18}} = \frac{1306028}{58739} = 22{\cdot}234$$

By this method may be constructed columns 2 and 7 of Table 3, showing the value of an immediate annuity of £1 during life, according to the Manchester Unity Table of Mortality for 1866-70.

12. *To find the value of an annuity of* £1 *deferred* n *years and then to continue so long as* x *shall live.*

The value of the annuity is the sum of the present values of £1 due at the end of $n+1$, $n+2$, $n+3$, . . . years, to the last tabular age, if x shall live so long.

Let ${}_n|a_x$ denote the present value of an annuity of £1 deferred n years and then to continue so long as x shall live; then

$${}_n|a_x = \frac{{}^{n+1}l_x v^{n+1} + {}^{n+2}l_x v^{n+2} + \ldots}{l_x}$$
$$= \frac{{}^{n+1}l_x v^{x+n+1} + {}^{n+2}l_x v^{x+n+2} + \ldots}{l_x v^x} \ldots\ldots (7).$$

The number in column N of Table II., opposite the age $x+n$, is, by Art. 11, the numerator of the fraction, and the number in column D, opposite the age x, is, by Art. 10, the denominator;

therefore
$${}_n|a_x = \frac{N_{x+n}}{D_x} \ldots\ldots (8).$$

Example 3. Value of an annuity of £1 to be entered upon at the expiration of 47 years on the life aged 18.

$$\frac{N_{65}}{D_{18}} = \frac{58956{\cdot}2}{58739} = 1{\cdot}004$$

* The method employed in the construction of the columns D and N of Table II. is described in the Appendix.

By this method may be constructed column 4 of Table III., showing the value of a deferred annuity of £1, to commence after age 65 and then to continue during life, according to the Manchester Unity Table of Mortality for 1866-70.

13. *To find the value of a temporary annuity of* £1 *for the next* n *years if* x *shall live so long.*

The value of the annuity is the sum of the present values of £1 due at the end of 1, 2, 3, . . . n years if x shall live so long.

Let ${}_{n}|a_x$ denote the present value of a temporary annuity of £1 for the next n years if x shall live so long; then since the value of a temporary annuity for the next n years, together with that of an annuity deferred n years, is equal to the value of an annuity during life,

$${}_{n}|a_x + {}_{n}|a_x = a_x;$$

by transposition

$${}_{n}|a_x = a_x - {}_{n}|a_x;$$

therefore

$${}_{n}|a_x = \frac{N_x}{D_x} - \frac{N_{x+n}}{D_x} = \frac{N_x - N_{x+n}}{D_x} \quad \ldots\ldots (9).$$

Example 4. Value of an annuity of £1 for the next 47 years on the life aged 18.

$$\frac{N_{18} - N_{65}}{D_{18}} = \frac{1306028 - 58956 \cdot 2}{58739} = 21 \cdot 231$$

This added to 1·004, the value of an annuity of £1 to be entered upon at the expiration of 47 years on the life aged 18 (Example 3), gives 22·235, the value of an annuity of £1 on the life aged 18 (Example 2).

By this method may be constructed column 3 of Table III., showing the value of a temporary annuity of £1 for a term of years if the life shall be in existence so long, according to the Manchester Unity Table of Mortality for 1866-70.

14. *To find the annual premium for an annuity of* £1 *deferred* n *years and then to continue so long as* x *shall live.*

The first annual premium made by members of Friendly Societies is generally completed at the end of the year, and the remaining premiums at the end of 2, 3, . . . n years: so that if the annual premium be £1, the present value of it, at yearly interest, will be ${}_{n}|a_x$, the value of a temporary annuity of £1 for the next n years on the life x.

Let $P_{n|}a_x$ denote the annual premium for an annuity of £1 deferred n years on the life x; then the value of a temporary annuity of £1 for n years on the life x is to £1 as the value of an annuity of £1 deferred n years on the life x is to its annual premium, or

$$|_n a_x \;:\; 1 \;::\; {}_{n|}a_x \;:\; \frac{{}_{n|}a_x}{|_n a_x},$$

$$\frac{N_x - N_{x+n}}{D_x} \;:\; 1 \;::\; \frac{N_{x+n}}{D_x} \;:\; \frac{N_{x+n}}{N_x - N_{x+n}};$$

therefore

$$P_{n|}a_x = \frac{N_{x+n}}{N_x - N_{x+n}} \quad \ldots\ldots (10).$$

Example 5. Annual premium for an annuity of £1 to be entered upon at the expiration of 47 years on the life aged 18.

$$\frac{N_{65}}{N_{18} - N_{65}} = \frac{58956{\cdot}2}{1306028 - 58956{\cdot}2} = {\cdot}04728$$

By this method may be constructed column 5 of Table III., showing the annual premium for a deferred annuity of £1, to commence after age 65 and then to continue during life, according to the Manchester Unity Table of Mortality for 1866-70.

TABLE II. *Preparatory Table for finding the Values of Annuities, according to the Manchester Unity Table of Mortality for* 1866-70. *Interest* 3 *per Cent.*

Age.	D_x	N_x	Age.	D_x	N_x
18	58739	1306028	60	9919·2	99879·5
19	56709	1249319	61	9323·1	90556·4
			62	8739·0	81817·4
20	54738	1194581	63	8168·6	73648·8
21	52823	1141758	64	7614·5	66034·3
22	50964	1090794			
23	49160	1041634	65	7078·1	58956·2
24	47410	994224	66	6560·7	52395·5
			67	6063·8	46331·7
25	45712	948512	68	5584·3	40747·4
26	44066	904446	69	5119·9	35627·5
27	42471	861975			
28	40926	821049	70	4669·7	30957·8
29	39430	781619	71	4233·5	26724·3
			72	3811·9	22912·4
30	37982	743637	73	3410·8	19501·6
31	36582	707055	74	3027·9	16473·7
32	35227	671828			
33	33916	637912	75	2668·4	13805·3
34	32645	605267	76	2334·1	11471·2
			77	2026·1	9445·1
35	31413	573854	78	1744·1	7701·0
36	30216	543638	79	1487·7	6213·3
37	29053	514585			
38	27923	486662	80	1256·8	4956·5
39	26826	459836	81	1050·2	3906·3
			82	867·40	3038·90
40	25762	434074	83	707·46	2331·44
41	24730	409344	84	569·20	1762·24
42	23729	385615			
43	22759	362856	85	450·72	1311·52
44	21819	341037	86	351·57	959·95
			87	269·50	690·45
45	20908	320129	88	202·90	487·55
46	20024	300105	89	149·89	337·66
47	19169	280936			
48	18338	262598	90	108·60	229·06
49	17531	245067	91	77·057	152·003
			92	53·522	98·481
50	16745	228322	93	36·348	62·133
51	15979	212343	94	24·106	38·027
52	15233	197110			
53	14506	182604	95	15·563	22·4644
54	13798	168806	96	9·7801	12·6843
			97	5·9701	6·7142
55	13108	155698	98	3·5329	3·1813
56	12438	143260	99	2·0366	1·1447
57	11785	131475			
58	11149	120326	100	1·1447	
59	10527	109798·7			

TABLE III. *Value of an Annuity of* £1, *according to the Manchester Unity Table of Mortality for* 1866-70. *Interest* 3 *per Cent.*

Age.	Immediate.	Temporary.	Deferred.		Age.	Immediate.
	During Life.	To continue until Age 65.	To commence after Age 65.	Annual Premium.		During Life.
	£	£	£	£		£
18	22·234	21·231	1·004	·04728	60	10·069
19	22·030	20·991	1·040	·04953	61	9·713
					62	9·362
20	21·824	20·747	1·077	·05192	63	9·016
21	21·615	20·499	1·116	·05445	64	8·672
22	21·403	20·246	1·157	·05714		
23	21·189	19·989	1·199	·06000	65	8·329
24	20·971	19·727	1·244	·06304	66	7·986
					67	7·641
25	20·750	19·460	1·290	·06628	68	7·297
26	20·525	19·187	1·338	·06973	69	6·959
27	20·296	18·907	1·388	·07342		
28	20·062	18·621	1·441	·07736	70	6·630
29	19·823	18·328	1·495	·08158	71	6·313
					72	6·011
30	19·579	18·026	1·552	·08611	73	5·718
31	19·328	17·716	1·612	·09097	74	5·441
32	19·071	17·398	1·674	·09620		
33	18·809	17·070	1·738	·10183	75	5·174
34	18·541	16·735	1·806	·10792	76	4·915
					77	4·662
35	18·268	16·391	1·877	·11450	78	4·415
36	17·992	16·041	1·951	·12164	79	4·176
37	17·712	15·683	2·029	·12940		
38	17·429	15·317	2·111	·13784	80	3·944
39	17·141	14·944	2·198	·14707	81	3·720
					82	3·503
40	16·849	14·561	2·288	·15717	83	3·296
41	16·553	14·169	2·384	·16826	84	3·096
42	16·251	13·766	2·485	·18048		
43	15·943	13·353	2·590	·19400	85	2·910
44	15·630	12·928	2·702	·20900	86	2·730
					87	2·562
45	15·311	12·492	2·820	·22574	88	2·403
46	14·987	12·043	2·944	·24448	89	2·253
47	14·656	11·580	3·076	·26559		
48	14·320	11·105	3·215	·28951	90	2·109
49	13·979	10·616	3·363	·31678	91	1·973
					92	1·840
50	13·635	10·114	3·521	·34810	93	1·709
51	13·289	9·599	3·690	·38436	94	1·577
52	12·940	9·069	3·870	·42674		
53	12·588	8·524	4·064	·47681	95	1·443
54	12·234	7·961	4·273	·53670	96	1·297
					97	1·125
55	11·878	7·380	4·498	·60942	98	·900
56	11·518				99	·562
57	11·156					
58	10·793					
59	10·430					

III. LIFE ASSURANCE.

15. A contract to secure a sum of money payable on the death of a person is called a *life assurance.*

16. *To find the value of an endowment assurance of* £1, *payable at the end of* n *years if* x *shall die in the* n*th year.*

Let $d_x = l_x - {}^1l_x$ denote the number of persons dying between the ages x and $x+1$, or in the 1st year, . . . ${}^{n-1}d_x = {}^{n-1}l_x - {}^nl_x$, the number dying between the ages $x+n-1$ and $x+n$, or in the nth year; then, by Art. 1, the probability of a person whose age is x dying in the nth year is $\frac{{}^{n-1}d_x}{l_x}$, and, by Art. 10, the present value of £1 payable at the end of n years if x shall die in the nth year is

$$\frac{{}^{n-1}d_x v^n}{l_x} \quad \ldots\ldots (1).$$

Example 1. Value of an endowment assurance of £10, payable at the end of 7 years if a person now aged 30 shall die in his 37th year, at 3 per cent., according to the Manchester Unity Table of Mortality for 1866-70.

$$\frac{d_{36}v^7}{l_{30}} \times 10 = \frac{844{\cdot}74 \times {\cdot}81309}{92193} = {\cdot}07450$$

By logarithms,

$$\begin{aligned} \log d_{36} &= 2{\cdot}926723 \\ \log v^7 &= \bar{1}{\cdot}910139 \\ \text{co log } l_{30} &= \bar{5}{\cdot}035302 \\ \log 10 &= 1{\cdot}000000 \\ \log &= \bar{2}{\cdot}872164 \\ &= {\cdot}07450 \end{aligned}$$

In formula (1) the numerator of the fraction is the present value of the number of pounds falling due at the end of n years, and the denominator the number now living to contribute equally the present fund.

If (1) be transformed by multiplying both numerator and denominator by v^x, by which the value of the fraction is not altered, it becomes

$$\frac{{}^{n-1}d_x v^{x+n}}{l_x v^x} \quad \ldots\ldots (2),$$

which is a more convenient formula for constructing tables of the values of assurances (presently to be explained) than formula (1).

The number in column C of Table IV., opposite each age, is the product of the number dying in the year following that age into the present value of £1 due as many years hence as are equal to that age increased by unity, so that in (2) the numerator of the fraction is equal to C_{x+n-1}, and the denominator to D_x. Hence the formula becomes finally

$$\frac{C_{x+n-1}}{D_x} \quad \ldots\ldots (3).$$

By means of columns C and D we are now enabled to compute the values of endowment assurances by simple division.

For example, as above: Value of an endowment assurance of £10, payable at the end of seven years if a person now aged 30 shall die in his 37th year, at 3 per cent.

$$\frac{C_{36}}{D_{30}} = \frac{282{\cdot}97}{37982} \times 10 = {\cdot}07450$$

17. *To find the value of an assurance of* £1, *payable at the end of the year in which* x *shall die.*

The value of the assurance is the sum of the present values of £1 payable at the end of 1, 2, 3, . . . years, to the last tabular age, if x shall die in the 1st, 2nd, 3rd, . . . years.

Let A_x denote the present value of an assurance of £1, payable at the end of the year in which x shall die; then, by Art. 16,

$$A_x = \frac{d_x v}{l_x} + \frac{{}^1d_x v^2}{l_x} + \ldots + \frac{{}^{z-1}d_x v^z}{l_x}$$

$$= \frac{d_x v + {}^1d_x v^2 + \ldots + {}^{z-1}d_x v^z}{l_x} \quad \ldots\ldots (4).$$

In formula (4) the numerator of the fraction is the sum of the present values of the number of pounds falling due at the end of 1, 2, 3, . . . years, to the last tabular age, and the denominator the number now living to contribute equally the present fund.

If (4) be transformed by multiplying both numerator and denominator by v^x, by which the value of the fraction is not altered, the equation becomes

$$A_x = \frac{d_x v^{x+1} + {}^1d_x v^{x+2} + \ldots + {}^{z-1}d_x v^{x+z}}{l_x v^x} \quad \ldots\ldots (5).$$

The number in column M of Table IV., opposite each age, is the sum of the numbers in column C opposite that and all the older ages;* thus

$M_x = C_x + C_{x+1} + C_{x+2} + \ldots$ to the last tabular age;

$M_x = C_x + M_{x+1}$;

therefore

$$A_x = \frac{M_x}{D_x} \quad \ldots\ldots (6).$$

Example 2. Value of an assurance of £1 on the life aged 30.

$$\frac{M_{30}}{D_{30}} = \frac{15217{\cdot}66}{37982} = {\cdot}40065$$

18. *To find the annual premium for an assurance of* £1, *payable at the end of the year in which* x *shall die.*

If the annual premium completed at the end of the year be £1, the present value of it, at yearly interest, will be a_x, the value of an annuity of £1 on the life x.

Let $P_x A_x$ denote the annual premium for an assurance of £1 on the life x; then the value of an annuity of £1 on the life x is to £1 as the value of an assurance of £1 on the life x is to its annual premium, or

$$a_x : 1 :: A_x : \frac{A_x}{a_x};$$

$$\frac{N_x}{D_x} : 1 :: \frac{M_x}{D_x} : \frac{M_x}{N_x};$$

therefore

$$P_x A_x = \frac{M_x}{N_x} \quad \ldots\ldots (7).$$

Example 3. Annual premium for an assurance of £1 on the life aged 30.

$$\frac{M_{30}}{N_{30}} = \frac{15217{\cdot}66}{743637} = {\cdot}02046$$

* The method employed in the construction of the columns C and M of Table IV. is described in the Appendix.

TABLE IV. *Preparatory Table for finding the Values of Assurances, according to the Manchester Unity Table of Mortality for* 1866-70. *Interest* 3 *per Cent.*

Age.	C_x	M_x	Age.	C_x	M_x
18	319·36	18991·35	60	307·23	6721·48
19	320·27	18671·99	61	312·68	6414·25
			62	315·65	6101·57
20	320·66	18351·72	63	316·27	5785·92
21	320·58	18031·06	64	314·71	5469·65
22	319·83	17710 48			
23	318 54	17390·65	65	311·18	5154·94
24	316 68	17072·11	66	305·81	4843·76
			67	303·00	4537·95
25	314·39	16755·43	68	301·73	4234·95
26	311·59	16441·04	69	301·09	3933·22
27	308·14	16129 45			
28	304·09	15821·31	70	300·17	3632·13
29	299·56	15517·22	71	298·38	3331·96
			72	290·19	3033·58
30	294·53	15217·66	73	283 56	2743·39
31	289·07	14923·13	74	271·33	2459·83
32	285 24	14634·06			
33	282·92	14348·82	75	256·62	2188·50
34	281·86	14065·90	76	240·01	1931·88
			77	223·01	1691·87
35	281·95	13784·04	78	205 61	1468·86
36	282·97	13502·09	79	187·91	1263 25
37	283·53	13219·12			
38	283·35	12935·59	80	170·06	1075·34
39	282·82	12652·24	81	152·17	905·28
			82	134·66	753·11
40	281·78	12369·42	83	117·70	618·45
41	280·31	12087·64	84	101·90	500·75
42	278 85	11807·33			
43	277·38	11528·48	85	86·040	398·853
44	275·89	11251·10	86	71·827	312·813
			87	58·766	240·986
45	274·32	10975 21	88	47·100	182·220
46	272·69	10700·89	89	36·956	135·120
47	272·42	10428·20			
48	273·32	10155·78	90	28·379	98·164
49	275·12	9882·46	91	21·300	69·785
			92	15·629	48·485
50	277·65	9607·34	93	11·200	32·856
51	280·70	9329·69	94	7·8284	21·6564
52	283·44	9048·99			
53	285·79	8765·55	95	5·3201	13·8280
54	287·71	8479·76	96	3·5156	8·5079
			97	2·2541	4·9923
55	289·15	8192 05	98	1·3995	2·7382
56	290·02	7902·90	99	·84106	1·33866
57	292·76	7612·88			
58	296·84	7320·12	100	·49760	·49760
59	301·80	7023·28			

TABLE V. *Value of an Assurance of* £1, *according to the Manchester Unity Table of Mortality for* 1866-70. *Interest* 3 *per Cent.*

Age.	Single Premium.	Annual Premium.	Age.	Single Premium.	Annual Premium.
	£	£		£	£
18	·32332	·01454	60	·67762	·06730
19	·32926	·01495	61	·68800	·07083
			62	·69820	·07458
20	·33526	·01536	63	·70831	·07856
21	·34135	·01579	64	·71832	·08282
22	·34751	·01624			
23	·35376	·01670	65	·72829	·08744
24	·36010	·01717	66	·73830	·09245
			67	·74837	·09794
25	·36654	·01766	68	·75837	·10393
26	·37310	·01818	69	·76822	·11040
27	·37978	·01871			
28	·38658	·01927	70	·77781	·11733
29	·39354	·01985	71	·78705	·12468
			72	·79582	·13240
30	·40065	·02046	73	·80432	·14068
31	·40794	·02111	74	·81239	·14932
32	·41542	·02178			
33	·42307	·02249	75	·82015	·15853
34	·43087	·02324	76	·82768	·16841
			77	·83504	·17913
35	·43880	·02402	78	·84219	·19074
36	·44685	·02484	79	·84913	·20331
37	·45500	·02569			
38	·46326	·02658	80	·85562	·21696
39	·47164	·02751	81	·86201	·23175
			82	·86824	·24782
40	·48014	·02850	83	·87418	·26527
41	·48878	·02953	84	·87974	·28416
42	·49759	·03062			
43	·50655	·03177	85	·88492	·30412
44	·51566	·03299	86	·88976	·32586
			87	·89420	·34903
45	·52493	·03428	88	·89808	·37375
46	·53440	·03566	89	·90146	·40017
47	·54401	·03712			
48	·55381	·03867	90	·90390	·42855
49	·56371	·04033	91	·90563	·45910
			92	·90589	·49233
50	·57374	·04208	93	·90393	·52880
51	·58387	·04394	94	·89838	·56950
52	·59404	·04591			
53	·60427	·04800	95	·88852	·61555
54	·61456	·05023	96	·86992	·67074
			97	·83622	·74754
55	·62497	·05261	98	·77506	·86072
56	·63538	·05516	99	·65730	1·1624
57	·64598	·05790			
58	·65657	·06084			
59	·66717	·06397			

IV. HEALTH ASSURANCE.

19. A contract to secure a weekly payment in sickness, incapacitating from ordinary occupation, is called a *health assurance*.

20. Tables of Sickness, showing from the experience of large numbers the amount of sickness to which the members of Friendly Societies are liable at each age, have been constructed by Mr. Ansell, Mr. Neison, Mr. Finlaison, and Mr. Ratcliffe, and published in their works on Friendly Societies (see list, page 2). Mr. Ratcliffe's Sickness Tables for 1866-70, constructed from the latest and largest experience, include all sickness experienced, and show the amount of sickness in various periods of sickness, as the Manchester Unity and many other Friendly Societies assure full sick benefits in a temporary period of sickness, and then make a reduction in a continuance of the same sickness.

21. *To find the value, at age* x, *of an assurance of* £1 *per week in the first twelve months' sickness during life.*

Let s_x denote the number of days' sickness experienced in the year following the age x according to any sickness table, and, assuming that the sickness in the year is uniformly distributed through the year, let $\frac{s_x}{365}$ denote the number of cases of sickness on any day of the year.

Let l_x denote the number of persons whose age is x living at the beginning of the year, and, assuming that the deaths in the year are uniformly distributed through the year, let $l_{x'}$, $l_{x''}$, $l_{x'''}$, . . . denote the numbers living to the end of the 1st, 2nd, 3rd, . . . days of the year.

Then the probability of a person whose age is x being sick on the first day of the year is $\frac{s_x}{365 l_x}$.

As each of the $l_{x'}$ persons living on the second day of the year has the same chance of being one of those who are sick, the probability of x' being sick on the second day is $\frac{s_x}{365 l_{x'}}$; but

as only $l_{x'}$ persons survive the first day, the probability of x living a day, so as to be liable to sickness on the second day, is $\frac{l_{x'}}{l_x}$. Therefore the probability, at the beginning of the year, of x being alive and sick on the second day of the year is $\frac{s_x}{365 l_{x'}} \times \frac{l_{x'}}{l_x} = \frac{s_x}{365 l_x}$, which is the same probability as x being sick on the first day of the year.

As each of the $l_{x''}$ persons living on the third day of the year has the same chance of being one of those who are sick, the probability of x'' being sick on the third day is $\frac{s_x}{365 l_{x''}}$; but as only $l_{x''}$ persons survive the second day, the probability of x living two days, so as to be liable to sickness on the third day, is $\frac{l_{x''}}{l_x}$. Therefore the probability, at the beginning of the year, of x being alive and sick on the third day of the year is $\frac{s_x}{365 l_{x''}} \times \frac{l_{x''}}{l_x} = \frac{s_x}{365 l_x}$, which is the same probability as x being sick on the first day of the year.

And the probability, at the beginning of the year, of x being alive and sick on any day of that year is always $\frac{s_x}{365 l_x}$.

And the sum of the probabilities, at the beginning of the year, of x being alive and sick on each day of that year is $\frac{s_x}{365 l_x} \times 365 = \frac{s_x}{l_x}$, that is the average number of days' sickness to each person in the year following the age x.

The present value, at the beginning of the year, of £1 payable

on the first day of the year if x be then alive and sick, is . . . . . $\frac{s_x}{365 l_x} \cdot 1$

on the second day, is equal to the present value of $\frac{s_x}{365 l_x}$ pounds due 1 day hence, that is . . . . . $\frac{s_x}{365 l_x} \cdot \frac{1}{1+\frac{i}{365}}$

on the third day, is equal to the present value of $\frac{s_x}{365 l_x}$ pounds due 2 days hence, that is . . . . . $\Big\}$ $\frac{s_x}{365 l_x} \cdot \frac{1}{1+\frac{2i}{365}}$

on the 365th day, is equal to the present value of $\frac{s_x}{365 l_x}$ pounds due 364 days hence, that is . . . . . $\Big\}$ $\frac{s_x}{365 l_x} \cdot \frac{1}{1+\frac{364i}{365}}$

The sum of these values is equal to the product of $\frac{s_x}{365 l_x}$ into the sum of the series

$$1+\frac{1}{1+\frac{i}{365}}+\frac{1}{1+\frac{2i}{365}}+\ldots+\frac{1}{1+\frac{364i}{365}}.$$

The sum of the series is $\frac{365}{1+\frac{i}{2}}$ very nearly; therefore the value, at the beginning of the year, of £1 payable on each day of that year if x be then alive and sick, is $\frac{s_x}{365 l_x} \times \frac{365}{1+\frac{i}{2}} = \frac{s_x}{l_x} \times \frac{1}{1+\frac{i}{2}}$, that is the present value of as many pounds as are equal to the average number of days' sickness to each person in the year following the age x due half a year hence.

Let £$\frac{1}{7}$ denote the daily payment in sickness, then £$\frac{1}{7} \times \frac{s_x}{l_x} \times \frac{1}{1+\frac{i}{2}}$ is the value, at age x, of an assurance of £$\frac{1}{7}$ per day in sickness for the following year, or making $\frac{s_x}{l_x}$ to denote the average number of weeks' sickness yearly to each person, £ $\frac{s_x}{l_x} \cdot \frac{1}{1+\frac{i}{2}}$ is the value, at age x, of an assurance of £1 per week payable daily in sickness for the following year.

Let $\mathfrak{s}_x$ denote the value, at age x, of an assurance of £1 per week in sickness for the following year, $\mathfrak{s}_{x+n}$ the value, at age $x+n$, of the same payment for the following year; then

the value, at age x, of $\mathfrak{s}_x$ being . . . . . $\mathfrak{s}_x$

the value, at age x, of $\mathfrak{s}_{x+1}$ is equal to the present value of $\mathfrak{s}_{x+1}$ due one year hence if x be then alive, that is . . . . . $\Big\}$ $\mathfrak{s}_{x+1} \cdot \frac{{}^1l_x v}{l_x}$

the value, at age x, of s_{x+n} is equal to the present value of s_{x+n} due n years hence if x be then alive, that is . . . . . } $s_{x+n} \cdot \frac{{}^{n}l_x v^n}{l_x}$

Let S_x denote the value, at age x, of an assurance of £1 per week in sickness during life; then

$$S_x = s_x + s_{x+1}\frac{{}^{1}l_x v}{l_x} + s_{x+2}\frac{{}^{2}l_x v^2}{l_x} + \ldots + s_{x+z}\frac{{}^{z}l_x v^z}{l_x},$$

$$= \frac{s_x l_x + s_{x+1}{}^{1}l_x v + s_{x+2}{}^{2}l_x v^2 + \ldots + s_{x+z}{}^{z}l_x v^z}{l_x} \ldots\ldots (1).$$

In formula (1) the numerator of the fraction is the sum of the present values of the number of pounds falling due at the end of 0, 1, 2, . . . years, to the last tabular age, and the denominator the number now living to contribute equally the present fund.

If (1) be transformed by multiplying both numerator and denominator by v^x, by which the value of the fraction is not altered, the equation becomes

$$S_x = \frac{s_x l_x v^x + s_{x+1}{}^{1}l_x v^{x+1} + s_{x+2}{}^{2}l_x v^{x+2} + \ldots + s_{x+z}{}^{z}l_x v^{x+z}}{l_x v^x} \ldots\ldots (2).$$

The number in column C′ of Table VII., opposite each age, is the product of as many pounds as are equal to the average number of weeks' sickness to each person in the year following that age due half a year hence into the number opposite that age in column D of Table II.

The number in column M′ of Table VII., opposite each age, is the sum of the numbers in column C′ opposite that and all the older ages;* thus

$$M'_x = C'_x + C'_{x+1} + C'_{x+2} + \ldots \text{ to the last tabular age;}$$
$$M'_x = C'_x + M_{x+1};$$

therefore

$$S_x = \frac{M'_x}{D_x} \ldots\ldots (3).$$

Example 1. Value, at age 18, of an assurance of £1 per week in the first twelve months' sickness during life.

$$\frac{M'_{18}}{D_{18}} = \frac{1661800}{58739} = 28{\cdot}291$$

* The method employed in the construction of columns C′ and M′ of Table VII. is described in the Appendix.

22. *To find the value, at age* x, *of an assurance of* £1 *per week in the first twelve months' sickness for* n *years*

Let ${}_nS_x$ denote the sum of the first n terms of the series S_x, and ${}_n|S_x$ the sum of all the terms after the first n terms; then

$${}_nS_x + {}_n|S_x = S_x;$$

by transposition $${}_nS_x = S_x - {}_n|S_x;$$

therefore $${}_nS_x = \frac{M'_x}{D_x} - \frac{M'_{x+n}}{D_x} = \frac{M'_x - M'_{x+n}}{D_x} \quad \ldots\ldots (4).$$

Example 2. Value, at age 24, of an assurance of £1 per week in the first twelve months' sickness until arriving at age 65.

$$\frac{M'_{24} - M'_{65}}{D_{24}} = \frac{1440254 - 334968}{47410} = 23{\cdot}313$$

23. *To find the annual premium, at age* x, *for an assurance of* £1 *per week in the first twelve months' sickness during life.*

If the annual premium completed at the end of the year be £1, the present value of it, at yearly interest, will be a_x, the value of an annuity of £1 on the life x.

Let P_xS_x denote the annual premium, at age x, for an assurance of £1 per week in the first twelve months' sickness during life; then the value of an annuity of £1 on the life x is to £1 as the value of an assurance of £1 per week in the first twelve months' sickness during life is to its annual premium, or

$$a_x : 1 :: S_x : \frac{S_x}{a_x};$$

therefore $$P_xS_x = \frac{S_x}{a_x} \quad \ldots\ldots (5).$$

Example 3. Annual premium, at age 18, for an assurance of £1 per week in the first twelve months' sickness during life.

$$\frac{S_{18}}{a_{18}}, \begin{array}{l}\text{Example 1,} = \frac{28{\cdot}291}{22{\cdot}234} = 1{\cdot}2724\\ \text{Table III.,}\end{array}$$

24. *To find the annual premium, at age* x, *for an assurance of* £1 *per week in the first twelve months' sickness for* n *years.*

If the annual premium completed at the end of the year be £1, the present value of it, at yearly interest, will be ${}_na_x$, the value of a temporary annuity of £1 for the next n years if x shall live so long.

Let $P|_nS_x$ denote the annual premium, at age x, for an assurance of £1 per week in the first twelve months' sickness for n years; then the value of a temporary annuity of £1 for n years if x shall live so long is to £1 as the value of an assurance of £1 per week in the first twelve months' sickness for n years is to its annual premium, or

$$|_na_x : 1 :: |_nS_x : \frac{|_nS_x}{|_na_x};$$

therefore

$$P|_nS_x = \frac{|_nS_x}{|_na_x} \quad \ldots\ldots (6).$$

Example 4. Annual premium, at age 24, for an assurance of £1 per week in the first twelve months' sickness until arriving at age 65.

$$\frac{|_{41}S_{24}}{|_{41}a_{24}}, \text{ Example 2,} = \frac{23{\cdot}313}{19{\cdot}727} = 1{\cdot}1818$$
$$\text{Table III.,}$$

TABLE VI. *Average Amount of Sickness Yearly in the first Twelve Months' Sickness, and after Twelve Months' continued Sickness, to each Member of the Independent Order of Odd Fellows, Manchester Unity Friendly Society for* 1866-70, *in the Rural, Town, and City Districts combined.*

Age.	First Twelve Months.	After Twelve Months.	Age.	First Twelve Months.	After Twelve Months.
	weeks.	weeks.		weeks.	weeks.
18	·6599	·0021	60	2·6520	1·3812
19	·6753	·0068	61	2·8326	1·5665
			62	3·0332	1·7756
20	·6907	·0115	63	3·2536	2·0088
21	·7061	·0162	64	3·4940	2·2658
22	·7216	·0189			
23	·7309	·0256	65	3·7543	2·5467
24	·7402	·0297	66	4·0344	2·8518
			67	4·3092	3·2477
25	·7475	·0328	68	4·5785	3·7346
26	·7526	·0387	69	4·8425	4·3125
27	·7606	·0455			
28	·7716	·0524	70	5·1010	4·9814
29	·7854	·0590	71	5·3540	5·7413
			72	5·5852	6·5156
30	·8022	·0646	73	5·7944	7·3047
31	·8218	·0753	74	5·9818	8·1081
32	·8406	·0852			
33	·8587	·0935	75	6·1472	8·9263
34	·8759	·1052	76	6·2906	9·7591
			77	6·4109	10·6913
35	·8924	·1191	78	6·5081	11·7230
36	·9080	·1259	79	6·5821	12·8542
37	·9269	·1344			
38	·9493	·1463	80	6·6315	14·0864
39	·9750	·1577	81	6·6037	15·4721
			82	6·5861	16·7403
40	1·0042	·1705	83	6·5787	17·8910
41	1·0367	·1850	84	6·5815	18·9245
42	1·0729	·2037			
43	1·1128	·2267	85	6·5961	19·8369
44	1·1562	·2542	86	6·3172	20·9368
			87	6·0208	22·0542
45	1·2034	·2859	88	5·7309	23·1641
46	1·2541	·3220	89	5·4233	24·2907
47	1·3098	·3597			
48	1·3702	·3991	90	5·1063	25·4267
49	1·4355	·4403	91	5·1374	25·5676
			92	5·1685	26·9935
50	1·5057	·4830	93	5·2000	27·8040
51	1·5806	·5276			
52	1·6629	·5590			
53	1·7527	·6372			
54	1·8499	·7024			
55	1·9545	·7743			
56	2·0665	·8532			
57	2·1923	·9532			
58	2·3318	1·0746			
59	2·4850	1·2173			

The average number of weeks' sickness yearly to each person, and also in the first twelve months' sickness, for each age, from age 84 to 93, has been deduced from columns D and K [C'] of the *Supplementary Report.*

TABLE VII. *Preparatory Table for finding the values of Assurances in the first Twelve Months' Sickness, according to the Manchester Unity Tables of Sickness and Mortality for* 1866-70. *Interest* 3 *per Cent.*

Age.	$C'x$	$M'x$	Age.	$C'x$	$M'x$
18	38189	1661800	60	25917	465415
19	37730	1623611	61	26018	439498
			62	26115	413480
20	37248	1585881	63	26185	387365
21	36747	1548633	64	26212	361180
22	36232	1511886			
23	35400	1475654	65	26180	334968
24	34574	1440254	66	26077	308788
			67	25744	282711
25	33665	1405680	68	25190	256967
26	32674	1372015	69	24427	231777
27	31826	1339341			
28	31112	1307515	70	23468	207350
29	30511	1276403	71	22331	183882
			72	20976	161551
30	30019	1245892	73	19471	140575
31	29619	1215873	74	17844	121104
32	29174	1186254			
33	28693	1157080	75	16161	103260
34	28172	1128387	76	14466	87099
			77	12797	72633
35	27618	1100215	78	11183	59836
36	27031	1072597	79	9647·2	48653·4
37	26531	1045566			
38	26116	1019035	80	8211·6	39006·2
39	25769	992919	81	6832·5	30794·6
			82	5628·4	23962·1
40	25488	967150	83	4585·4	18333·7
41	25259	941662	84	3690·8	13748 3
42	25083	916403			
43	24952	891320	85	2929·1	10057·5
44	24854	866368	86	2188·1	7128·4
			87	1598 7	4940·3
45	24788	841514	88	1145·6	3341·6
46	24742	816726	89	800·86	2195·98
47	24736	791984			
48	24755	767248	90	546·34	1395·12
49	24793	742493	91	390·02	848·78
			92	272·54	458·76
50	24840	717700	93	186·22	186·22
51	24884	692860			
52	24957	667976			
53	25049	643019			
54	25148	617970			
55	25242	592822			
56	25322	567580			
57	25455	542258			
58	25614	516803			
59	25774	491189			

TABLE VIII. *Value of an Assurance of* £1 *per Week in the first Twelve Months' Sickness, according to the Manchester Unity Tables of Sickness and Mortality for* 1866-70. *Interest* 3 *per Cent.*

Age.	During Life.		To continue until Age 65.		Age.	During Life.	
	Single Premium.	Annual Premium.	Single Premium.	Annual Premium.		Single Premium.	Annual Premium.
	£	£	£	£		£	£
18	28·291	1·2724	22 589	1·0640	60	46·921	4·6599
19	28 631	1·2996	22·724	1·0826	61	47 141	4·8534
					62	47·314	5·0538
20	28·972	1·3275	22·853	1·1015	63	47·421	5·2596
21	29·317	1·3563	22·976	1·1208	64	47·433	5·4697
22	29·666	1·3861	23·093	1·1406			
23	30·017	1 4166	23·183	1·1598	65	47·325	5·6820
24	30·379	1·4486	23·313	1·1818	66	47·066	5·8936
					67	46·623	6·1017
25	30·751	1·4820	23·423	1·2036	68	46·016	6·3062
26	31·135	1·5169	23·533	1·2265	69	45 270	6 5052
27	31·535	1·5538	23 648	1·2508			
28	31·948	1·5925	23·764	1·2762	70	44·463	6·6973
29	32·371	1·6330	23·876	1·3027	71	43·435	6·8802
					72	42·381	7·0506
30	32·802	1·6754	23·983	1 3305	73	41·215	7·2079
31	33·237	1·7196	24·080	1·3592	74	39·996	7·3509
32	33·675	1·7658	24·166	1·3890			
33	34·116	1·8138	24·240	1·4200	75	38·697	7·4791
34	34·565	1·8642	24·304	1·4523	76	37·316	7·5922
					77	35·849	7·6896
35	35·024	1·9172	24·361	1·4862	78	34·308	7·7708
36	35·498	1·9730	24·412	1·5219	79	32·704	7·8314
37	35·988	2·0318	24·459	1·5596			
38	36·494	2·0939	24·498	1 5994	80	31·036	7·8692
39	37·013	2·1593	24·527	1·6413	81	29·323	7·8825
					82	27·625	7·8861
40	37·542	2·2281	24·539	1·6853	83	25·915	7·8626
41	38·078	2·3004	24 533	1·7315	84	24·154	7·8017
42	38·620	2·3765	24 503	1·7800			
43	39 163	2·4564	24·445	1·8307	85	22·314	7·6680
44	39·707	2·5404	24·355	1·8839	86	20·276	7·4271
					87	18·331	7·1550
45	40·248	2·6287	24·227	1·9394	88	16·469	6·8535
46	40·787	2·7215	24·059	1·9978	89	14·651	6·5029
47	41·316	2·8190	23·841	2·0588			
48	41·839	2·9217	23·573	2·1227	90	12·846	6 0910
49	42·353	3·0298	23·246	2·1897	91	11·015	5·5829
					92	8·571	4·6582
50	42·861	3·1435	22·856	2 2598	93	5·123	2·9977
51	43·361	3 2629	22 398	2·3334			
52	43·851	3·3888	21·861	2·4105			
53	44·328	3 5214	21·236	2·4913			
54	44·787	3·6609	20·510	2·5763			
55	45·226	3·8075	19·671	2·6654			
56	45·633	3 9619					
57	46·013	4·1245					
58	46·354	4·2948					
59	46·660	4·4736					

TABLE IX. *Preparatory Table for finding the Values of Assurances in Sickness after Twelve Months' continued Sickness, according to the Manchester Unity Tables of Sickness and Mortality for* 1866—70. *Interest* 3 *per Cent.*

Age.	C′	M′	Age.	C′	M′
18	121·53	714932·89	60	13498	508589
19	379·92	714811·36	61	14389	495091
			62	15288	480702
20	620·18	714431·44	63	16167	465414
21	843·08	713811·26	64	16998	449247
22	948·98	712968·18			
23	1239·9	712019·2	65	17759	432249
24	1387·3	710779·3	66	18433	414490
			67	19402	396057
25	1477·2	709392·0	68	20547	376655
26	1680·2	707914·8	69	21753	356108
27	1903·9	706234·6			
28	2112·8	704330·7	70	22918	334355
29	2292·0	702217 9	71	23947	311437
			72	24470	287490
30	2417·4	699925 9	73	24546	263020
31	2713 9	697508·5	74	24187	238474
32	2957·0	694794·6			
33	3124 3	691837·6	75	23467	214287
34	3383·5	688713·3	76	22442	190820
			77	21341	168378
35	3686·0	685329·8	78	20143	147037
36	3748·0	681643 8	79	18840	126894
37	3847 0	677895 8			
38	4024·8	674048·8	80	17442	108054
39	4168·0	670024·0	81	16008	90612
			82	14306	74604
40	4327·5	665856·0	83	12470	60298
41	4507·4	661528·5	84	10613	47828
42	4762·2	657021·1			
43	5083·2	652258·9	85	8808·9	37215·1
44	5464·4	647175·7	86	7252·1	28406·2
			87	5855·8	21154·1
45	5889·1	641711·3	88	4630·5	15298·3
46	6352·6	635822·2	89	3587·1	10667·8
47	6793·0	629469·6			
48	7210·5	622676·6	90	2720·5	7080·7
49	7604·7	615466·1	91	1941·1	4360·2
			92	1423·4	2419·1
50	7968·3	607861·4	93	995·70	995·70
51	8305·2	599893·1			
52	8339·7	591586·9			
53	9106·8	583197·2			
54	9548·5	574090·4			
55	9999·9	564541·9			
56	10455	554542			
57	11068	544087			
58	11804	533019			
59	12626	521215			

TABLE X. *Value of an Assurance of* £1 *per Week in Sickness after Twelve Months' continued Sickness, according to the Manchester Unity Tables of Sickness and Mortality for* 1866-70. *Interest* 3 *per Cent.*

Age.	During Life.		To continue until Age 65.		Age.	During Life.	
	Single Premium.	Annual Premium.	Single Premium.	Annual Premium.		Single Premium.	Annual Premium.
	£	£	£	£		£	£
18	12·171	·5474	4·813	·2267	60	51·273	5·0922
19	12·605	·5721	4·983	·2374	61	53·104	5·4673
					62	55·007	5·8756
20	13·052	·5981	5·155	·2485	63	56·976	6·3194
21	13 513	·6252	5·330	·2600	64	58 999	6·8033
22	13·990	·6536	5·508	·2721			
23	14·484	·6836	5·691	·2847	65	61·069	7·3321
24	14·992	·7149	5·875	·2978	66	63·178	7·9111
					67	65·315	8·5480
25	15 519	·7479	6·063	·3116	68	67·449	9·2434
26	16 065	·7827	6·256	·3261	69	69·554	9·9948
27	16·629	·8193	6·451	·3412			
28	17·210	·8578	6·648	·3570	70	71·601	10·7995
29	17·809	·8984	6·847	·3736	71	73·565	11·6529
					72	75·419	12·5468
30	18·428	·9412	7·047	·3909	73	77·114	13·4862
31	19·067	·9865	7·251	·4093	74	78·759	14·4751
32	19·723	1·0342	7·453	·4284			
33	20 399	1 0845	7·654	·4484	75	80·305	15·5209
34	21·097	1·1379	7·856	·4694	76	81·753	16·6334
					77	83·104	17·8258
35	21·817	1·1943	8·057	·4916	78	84·305	19·0951
36	22·559	1·2538	8·254	·5146	79	85·295	20·4250
37	23·333	1·3174	8·455	·5391			
38	24·140	1·3850	8·660	·5654	80	85·975	21·7989
39	24·977	1·4571	8·864	·5931	81	86·281	23·1938
					82	86·009	24·5530
40	25 846	1·5340	9 068	·6228	83	85·232	25·8592
41	26·750	1·6160	9 271	·6543	84	84·027	27·1405
42	27·689	1·7038	9·472	·6881			
43	28·659	1·7976	9·667	·7240	85	82·568	28·3739
44	29·661	1·8977	9 850	·7619	86	80·798	29·5963
					87	78·494	30·6378
45	30·692	2·0046	10·018	·8020	88	75·398	31·3766
46	31·753	2·1187	10·166	·8441	89	71·171	31·5894
47	32 838	2 2406	10·289	·8885			
48	33 956	2·3712	10·383	·9350	90	65·200	30·9151
49	35·107	2·5114	10·451	·9845	91	56·584	28·6792
					92	45·198	24·5641
50	36·301	2·6623	10·487	1·0369	93	27·394	16·0293
51	37·543	2·8251	10·492	1 0930			
52	38·836	3 0012	10·460	1·1534			
53	40·204	3 1938	10·406	1·2208			
54	41·607	3·4009	10·280	1·2913			
55	43·069	3·6259	10·093	1·3676			
56	44·584	3·8708					
57	46·168	4·1384					
58	47·809	4 4296					
59	49·512	4·7471					

VALUATION OF LIABILITIES.

25. A valuation of the policies at the end of any given period is called a *valuation of the liabilities.*

26. *To find the value of a policy.*

If the rates of sickness and mortality experienced and the rate of interest realised correspond with the rates of sickness, mortality, and interest adopted in computing the premiums, the fund in hand to provide for the claims under the policies is the premiums received at the end of each year less the claims by sickness and death at the end of each year, accumulated at compound interest. Thus the accumulated fund at the end of each year is the accumulation of the premiums received less the claims paid at the early ages to meet the claims which exceed the premiums at the advanced ages. The share of each person living at the end of each year is the value of each policy.

Let ${}_nV_x$ denote the present value of a policy for £1 on the life of a person of the age x at entry after it has been n years in force, the premium being due a year hence, A_x the value of the sum assured on the life x, and P_x its risk annual premium; then at the end of a year the value of the policy will be equal to the value of the sum assured on the life $x+1$ less the value of the future premiums, that is

$$ {}_1V_x = A_{x+1} - P_x {}^1a_x ;$$

at the end of two years

$$ {}_2V_x = A_{x+2} - P_x {}^2a_x ;$$

at the end of n years

$$ {}_nV_x = A_{x+n} - P_x {}^na_x .$$

Rule. *The value of a policy is the value of the sum assured at the present age less the value of the future premiums, computed by the rates of sickness and mortality adopted, and the rate of interest which will most likely be realised in the future.**

If the accumulated fund in hand is equal to the sum of the values of the sums assured less the sum of the values of the future premiums the Society is solvent.

* It is impossible to obtain correct rates of Sickness and mortality which may be expected to prevail in the future from the experience of a single Lodge with small numbers and financially independent as regards Health Assurance, but this is no reason why an incorrect rate of interest should be adopted in valuing the liabilities.

APPENDIX.

To construct a table of the logarithms of the present value of £1 *due at the end of any number of years.*

The present value of £1 due a year hence is $\frac{1}{1+i}$, the logarithm of which is the arithmetical complement of log $(1+i)$. When the rate is 3 per cent. log $(1+i)$=log 1·03= 0·01283,72247, the arithmetical complement of which is $\bar{1}$·98716,27753. By taking the first nine multiples of this logarithm may be constructed a table of the logarithms of the present value of £1 due at the end of any number of years from 1 to 100 which will be true to the last figure to six places of decimals, thus:

Logarithm of the present value of £1.

Years.	
1	$\bar{1}$·987162 775
2	·974325 550
3	·961488 325
4	·948651 100
5	·935813 875
*	*

To construct columns D *and* N.

Since $D_x = l_x v^x$;

therefore $\log D_x = \log l_x + \log v^x$.

Log D_x is formed in reverse order to facilitate the formation of column N, thus:

	Log D_x	D_x and N_x	
Log l_{100}	1·342423		
,, v^{100}	$\bar{2}$·716278		
100	0·058701	1·1447	99
	1·579784		
	$\bar{2}$·729115		
99	0·308899	2·0366	
	1·806180	$\overline{3}$·1813	98
	$\bar{2}$·741952		
98	0·548132	3·5329	
		$\overline{6}$·7142	97
*	*	*	*

To construct columns C *and* M.

Since $C_x = d_x v^{x+1}$;

therefore $\text{Log } C_x = \log d_x + \log v^{x+1}$.

	Log C_x	C_x and M_x	
Log d_{100}	0·993441		
,, v^{101}	$\bar{2}$·703440		
100	$\bar{1}$·696881	·49760	100
	1·208549		
	$\bar{2}$·716278		
99	$\bar{1}$·924827	·84106	
	1·416857	$\overline{1}$·33866	99
	$\bar{2}$·729115		
98	0·145972	1·3995	
		$\overline{2}$·7382	98
	*	* *	*

To construct columns C′ *and* M′.

Since $C'_x = s'_x w D_x$, where s'_x denotes the average number of weeks' sickness to each person in the year following the age x, and w the present value of £1 due half a year hence;

therefore $\log C'_x = \log s'_x + \log w + \log D_x$.

The present value of £1 due half a year hence at 3 per cent. is $\frac{1}{1+\frac{\cdot 03}{2}} = \frac{1}{1\cdot 015}$, the logarithm of which is the arithmetical complement of log 1·015. Log 1·015 = 0·00646,60422, the arithmetical complement of which is $\overline{1}$·99353,39578. Writing this logarithm to six places of decimals at the bottom of a card to be added to the other two logarithms at each age, the formation is as follows:

	Log C'_x	C'_x and M'_x	
	[$\overline{1}$·993534]		
Log s'_{93}	0·716003		
,, D_{93}	1·560486		
93	2·270023	186·22	93
	0·713365		
	1·728531		
92	2·435430	272·54	
	0·710743	458·76	92
	1·886809		
91	2·591086	390·02	
		848·78	91
	*	* *	*

ADVERTISEMENT.

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The Mortality Table as printed on pages 5 and 6, having been made the basis of computation in the *Supplementary Report*, has been made the basis of computation in this Treatise, to admit of comparison of the deduced results of this Treatise with the corresponding results of the *Supplementary Report*.

Should a larger Treatise on Friendly Societies be called for a Chapter on the Construction of Tables of Mortality will be given, and also a method for verifying the columns N and M.